APPLICATION OF MOMENT-METHODS TO ELECTROMAGNETIC BIOLOGICAL IMAGING

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ABSTRACT

An explicit solution has been obtained for the dielectric properties of an inhomogeneous dielectric scatterer in terms of measured values of the incident and externally-scattered fields. Effects of secondary media, localized sources, and other means for improving the numerical stability will be described.

Introduction

We have previously used a pulse-function basis with point-matching to solve the volume electric field integral equation to determine the energy absorbed and its distribution in an inhomogeneous block model of man exposed to plane waves or near fields. More recently we have explored the possibility that moment-method solutions could be used effectively in the inverse problem, thereby allowing calculation of inhomogeneous dielectric properties for biomedical imaging. The matrix obtained for the direct problem has been found to permit a factorization such that it is possible to obtain an explicit solution for the dielectric properties of the N cells of a scatterer in terms of measured values of the incident and externally-scattered fields.

Derivation of Matrix Equation

The matrix equation relating incident field $\vec{E}_1^i$ to the internal field $\vec{E}_1$ within the N cells of a dielectric scatterer may be written as

$$\overleftrightarrow{A} \cdot \vec{E}_1 = -\vec{E}_1^i \tag{1}$$

The 3N by 3N matrix $\overleftrightarrow{A}$ permits the factorization

$$\overleftrightarrow{A} = \overleftrightarrow{A}_1 \cdot \overleftrightarrow{R} - \overleftrightarrow{I} \tag{2}$$

where $\overleftrightarrow{I}$ is the identity matrix, $\overleftrightarrow{A}_1$ depends only on frequency, cell size, and cell location, and $\overleftrightarrow{R}$ is a diagonal matrix containing only the values of $(\varepsilon_r - 1)$ for the various cells where ε_r is the complex permittivity ratio relative to free space.

Fields within the dielectric scatterer are related to the fields $\vec{E}_2^s$ scattered to N points outside the scatterer by

$$\vec{E}_2^s = \overleftrightarrow{B} \cdot \vec{E}_1 \tag{3}$$

But matrix $\overleftrightarrow{B}$ permits the factorization

$$\overleftrightarrow{B} = \overleftrightarrow{B}_1 \cdot \overleftrightarrow{R} \tag{4}$$

where $\overleftrightarrow{B}_1$ depends only on frequency, cell size, cell location, and location of the external measurement points.

Using the four equations given above, we may obtain

$$\overleftrightarrow{R}^{-1} \cdot \overleftrightarrow{B}_1^{-1} \cdot \vec{E}_2^s = \overleftrightarrow{A}_1 \cdot \overleftrightarrow{B}_1^{-1} \cdot \vec{E}_2^s + \vec{E}_1^i \tag{5}$$

Equation 5 allows explicit computation of $(\varepsilon_r - 1)$ for the various cells by taking ratios of the corresponding elements in two column vectors. Similar but somewhat more complicated expressions have been obtained for more realistic situations allowing a mixture of cells such that the unknown dielectric scatterer is in the presence of a body having known dielectric properties, or the source is tightly coupled to the scatterer.

Conclusions from Numerical Tests

Numerical testing of Eq. 5 with simple models has shown the instability generally found in inverse problems. Fractional errors in calculated dielectric properties are much greater than the fraction of noise allowed in the field measurements. The sensitivity to error increases with distance of the measurement points from the scatterer. Some improvement is obtained by making the field measurments within a layer of water or other dielectric having properties similar to those of the scatterer. Reduced sensitivity to noise may also be obtained by using localized sources.

We are currently seeking optimum antenna configurations for minimum error in the predicted dielectric properties. We will also use the generalized inverse to examine the effects of overdetermination (redundant data) and underdetermination on variance and resolution.